

Muon Magnetic Moment and the Goldstone Boson Higgs

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Abstract

We compute the correction to the muon magnetic moment in theories where the Higgs is a pseudo-Goldstone boson and leptons are partially composite. Using a general effective lagrangian we show that in some regions of parameters a sizable new physics contribution to the magnetic moment can be obtained from composite fermions that could explain the 3.5σ experimental discrepancy from the Standard Model prediction. This effect depends on the derivative interactions of the Higgs that do not modify the coupling of the Higgs to leptons and it does not require extremely light fermions, allowing to easily avoid LHC bounds. Our derivations can be in general applied to dipole operators in theories with Goldstone boson Higgs.

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1 Introduction

In this note we study new physics contributions to the muon anomalous magnetic moment in theories where the Higgs is a Goldstone boson (GB) and leptons are partially composite, see [1] for a review. These models are strongly motivated by the hierarchy problem because the Higgs boson, being a composite state, is not sensitive to scales much shorter than its size. This points to a scale of compositeness around TeV that is being tested at the LHC.

Our main motivation here is the long standing muon magnetic moment anomaly

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (2.8 \pm 0.8) \times 10^{-9} \quad (1)$$

($a_\mu = (g_\mu - 2)/2$) whose size suggests a new physics contribution of the order of the Standard Model (SM) electro-weak correction. In renormalizable theories where SM fields mix with heavy leptons the contribution scales as

$$\Delta a_\mu \sim \frac{g_\psi^2}{(4\pi)^2} \frac{m_\mu^2}{\Lambda^2} \quad (2)$$

where Λ is a new physics scale associated with the heavy fermions and g_ψ their coupling to the Higgs. At face value the effect is typically too small unless the fermions are as light as 200-300 GeV, in agreement with explicit models [2, 3, 4]. In theories with GB Higgs new diagrams arise from the non-linearities of the theory demanded by the symmetries and also UV contributions from the composite sector dynamics are expected. We wish to show that the size of these effects could account in certain regions of parameters for the anomaly (1), compatibly with bounds from flavor physics, LHC searches and electro-weak precision tests.

2 Partially Composite Muon

We work within the framework of composite Higgs models with partial compositeness. The Higgs is a GB of some strongly coupled theory with global symmetry G spontaneously broken to a subgroup H at a scale $f > v$. For concreteness we will focus on the minimal models based on $SO(5)/SO(4)$ but our results can be extended to other patterns of symmetry breaking and different representations, see [5]. SM fermions are partially composite, mixing with states of equal quantum numbers under the SM gauge symmetries.

The lagrangian for the composite states can be described in the most general fashion using the CCWZ formalism [6]. We focus here on new composite fermions and do not include vector resonances for simplicity and because they are typically heavier. Composite states are classified according to their representation under the unbroken group. The most general lagrangian compatible with the symmetries can be constructed with the aid of the connections e_μ and d_μ by writing down all possible invariants under the unbroken group. The connections are explicitly reported in appendix A for the coset $SO(5)/SO(4)$. Elementary fields can be introduced assigning them to a representation of $SO(5)$ and writing the most general couplings to the composite states using the GB matrix U .

For concreteness we study in detail the scenario where the left and right chirality of the muon couple to composite fermions in the $\mathbf{5}$ of $SO(5)$ but we provide the tools for computing in general dipole moments in models with GB Higgs. For the top quark, a model with the same structure can be found in [7], see also [8]. We refer to [7] and appendix A for details on the notation. We focus on a single generation and comment on the flavor structure in Sect. 4. The composite states decompose into a quadruplet and a singlet under $SO(4)$,

$$\mathbf{5} = \mathbf{4} + \mathbf{1} : \quad \psi_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} i(E_{-2} - N) \\ E_{-2} + N \\ i(E_{-1} + E) \\ E - E_{-1} \end{pmatrix}, \quad \psi_1 = \tilde{E} \quad (3)$$

with lagrangian

$$\begin{aligned} \mathcal{L}_{comp} = & \bar{\psi}_4(i \not{D} - m_4) \psi_4 + \bar{\psi}_1(i \not{D} - m_1) \psi_1 \\ & + i d_\mu^{\hat{a}} \left[c_L \bar{\psi}_{4L}^{\hat{a}} \gamma^\mu \psi_{1L} + c_R \bar{\psi}_{4R}^{\hat{a}} \gamma^\mu \psi_{1R} + h.c. \right] \end{aligned} \quad (4)$$

where

$$\begin{aligned} D_\mu \psi_1 &= [\partial_\mu + i g' B_\mu] \psi_1 \\ D_\mu \psi_4 &= [\partial_\mu - i e_\mu + i g' B_\mu] \psi_4 \\ d_\mu^{\hat{a}} &= \frac{\sqrt{2}}{f} D_\mu \pi^{\hat{a}} + \dots, \end{aligned} \quad (5)$$

g' and B_μ are the SM hypercharge coupling and field and $\pi^{\hat{a}}$ are the four components of the Higgs doublet. The second line in (4) contains the leading derivative interactions of the Higgs

with the fermions (controlled by the symmetry breaking scale f) that are characteristic of GB theories. These will play a crucial role in the computation of Δa_μ . Based on general power counting arguments we assume $c_{L,R}$ to be of order one [9]. The derivative couplings are in general complex unless the composite sector respects CP . Moreover $c_L = c_R$ if parity is preserved.

The mixing with the elementary fermions is given by,

$$-\mathcal{L}_{mixing} = y_{L_4} f (\bar{l}_L^{\mathbf{5}})^I U_{I\hat{a}} \psi_4^{\hat{a}} + y_{L_1} f (\bar{l}_L^{\mathbf{5}})^I U_{I5} \psi_1 + y_{R_4}^* f (\bar{\mu}_R^{\mathbf{5}})^I U_{I\hat{a}} \psi_4^{\hat{a}} + y_{R_1}^* f (\bar{\mu}_R^{\mathbf{5}})^I U_{I5} \psi_1 + h.c. \quad (6)$$

where

$$l_L^{\mathbf{5}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\nu_L \\ \nu_L \\ i\mu_L \\ \mu_L \\ 0 \end{pmatrix}, \quad \mu_R^{\mathbf{5}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mu_R \end{pmatrix}. \quad (7)$$

Diagonalizing the mass matrix one finds the following expression for the muon mass

$$m_\mu \approx \frac{f^2}{\sqrt{2}} \left[\frac{y_{L_4} y_{R_4}}{m_4} - \frac{y_{L_1} y_{R_1}}{m_1} \right] s_h c_h \quad (8)$$

valid to leading order in the mixings. We recall that the trigonometric dependence ($s_h \equiv \sin h/f$, $c_h \equiv \cos h/f$) is determined by the representations of the global symmetry. One can always choose the phases so that m_μ is real and we will assume this choice in the rest of the paper.

2.1 Contributions to a_μ

We parametrize the dipole moment operator of the muon as

$$\frac{X_\mu}{4m_\mu} \bar{\mu}_L \sigma^{\mu\nu} \mu_R e F_{\mu\nu} + h.c. \quad (9)$$

For m_μ real, $a_\mu = \text{Re}[X_\mu]$, while the imaginary part contributes to the electric dipole moment (EDM).

At 1-loop the new physics contribution to X_μ arises from diagrams with heavy fermions χ in the loop with charge -2, -1 or 0 and SM gauge fields or Higgs. To leading order ΔX_μ is generated by diagrams with one left and one right mixing corresponding to the function \mathcal{G} in the expressions reported in the appendix B.

There are two classes of contributions drawn in Fig. 1. The first corresponds to diagrams with heavy composite fermions in the loop and W , Z or Higgs with non-derivative interactions. These are analogous to the ones considered in renormalizable theories with vector-like fermions [2, 3, 4, 10, 11] except that the couplings of the composite leptons have

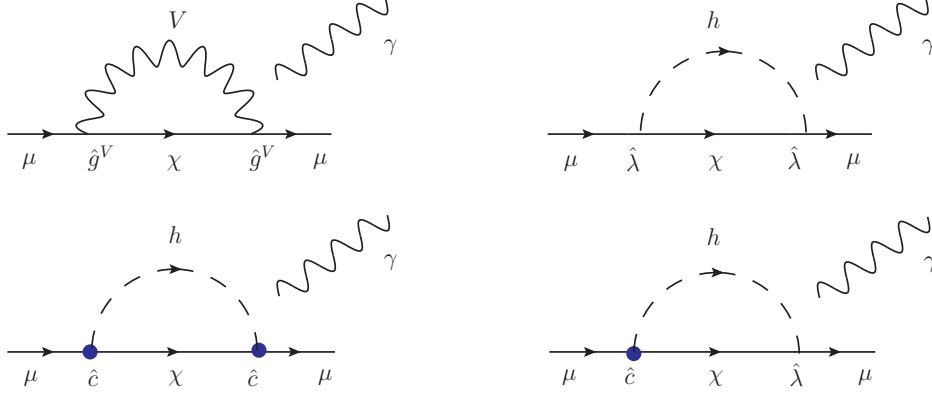


Figure 1: Diagrams contributing to Δa_μ . On the first line the diagrams with gauge and Yukawa interactions are shown while on the second line the ones with Higgs derivative interactions.

new contributions from the connections e_μ and d_μ . With the standard formulas collected in the appendix A, the contribution of heavy fermions coming from this first class of diagrams reads,

$$\begin{aligned}
\Delta X_\mu^Z &\simeq \frac{m_\mu m_\chi}{4\pi^2 v^2} (\hat{g}_L^Z) (\hat{g}_R^Z)^* \\
\Delta X_\mu^{W^-} &\simeq -\frac{m_\mu m_\chi}{8\pi^2 v^2} (\hat{g}_L^{W^-}) (\hat{g}_R^{W^-})^* \\
\Delta X_\mu^{W^+} &\simeq \frac{m_\mu m_\chi}{8\pi^2 v^2} (\hat{g}_L^{W^+}) (\hat{g}_R^{W^+})^* \\
\Delta X_\mu^h &\simeq \frac{1}{16\pi^2} \frac{m_\mu}{m_\chi} (\hat{\lambda}_L) (\hat{\lambda}_R)^*
\end{aligned} \tag{10}$$

($v = 246$ GeV) valid to first order in the mixings and in the limit $m_\chi \gg m_{Z,W,h}$. These contributions can be, in general, complex and generate both electric and magnetic dipole moments. Within an explicit model the couplings in the equation above are obtained by rotating the matrices of couplings to the mass basis. For this purpose, given the smallness of the muon mass, it is sufficient to use the rotation matrices to first order in the mixings. In the CCWZ parametrization this is particularly simple since the only off-diagonal terms in the mass matrix are the elementary-composite mixings. The contribution from Higgs exchange is not sub-leading contrary to the SM where it is suppressed by m_μ^2/m_h^2 compared to the gauge one. Note that in theories with vector-like leptons without GB structure the Higgs can have additional non-derivative interactions with the heavy fermions that dominate [10, 11].

The second type of contribution is strictly associated to the GB nature of the Higgs and is analogous to the one considered for dipole moment of baryons in QCD, see [12] and Refs.

therein. The term in the lagrangian (4) proportional to $c_{L,R}$ contains a derivative interaction of the Higgs with the composite fermions. Through this vertex two new diagrams can be drawn that contribute to the dipole moment shown on the second line of Fig. 1. We evaluate these new contributions in the appendix B. The loop diagrams are finite but their values depend on the regularization procedure. Evaluating the integrals in 4D one finds,

$$\begin{aligned}\Delta X_\mu^{(\partial h)^2} &\simeq -\frac{1}{48\pi^2} \frac{m_\mu m_\chi}{f^2} \hat{c}_L \hat{c}_R^* \\ \Delta X_\mu^{\partial hh} &\simeq \frac{1}{24\pi^2} \frac{m_\mu}{f} (\hat{c}_L \hat{\lambda}_R^* - \hat{\lambda}_L \hat{c}_R^*) ,\end{aligned}\tag{11}$$

valid within the same approximations as above.

Before analysing the explicit model above let us discuss the general structure of the result. The chiral structure of dipole moments is identical to the one of mass terms. As a consequence, the group theoretical structure, controlled by the global symmetries of the theory, is also similar in the two cases. To leading order the dipole moment must be proportional to the product of the mixings of left and right chirality of the muon. The Higgs dependence can be determined using a spurion analysis. To do this one should assign the elementary fields to a representation of the global symmetry and write all the invariants under the unbroken group using the GB matrix, see [5] for more details. One finds,

$$\Delta X_\mu = \sum_{A,i,j} x_A^{ij} y_L^i y_R^j (\bar{l}_L)^i U P_A^{ij} U^\dagger (\mu_R)^j \tag{12}$$

where $(l_L)^i$ and $(\mu_R)^j$ denote the embedding of the elementary fields into G representations \mathbf{r}_L^i and \mathbf{r}_R^j and P_A^{ij} are the projectors over the irreducible H representations contained in the product of $\mathbf{r}_L^i \times \mathbf{r}_R^j$. The coefficients x_A^{ij} contain the dynamical information.

When a single invariant exists, ΔX_μ will always be proportional to the muon mass because the Yukawa couplings have an identical expansion as eq. (12). For the model in eqs. (4),(6) this can be realised when $y_{L_4} = y_{L_1}$ and $y_{R_4} = y_{R_1}$ (other possibilities are $y_{L_1} = y_{R_1} = 0$ or $y_{L_4} = y_{R_4} = 0$). In this case one finds,

$$\Delta X_\mu \sim \frac{\kappa}{16\pi^2} \frac{m_\mu^2}{f^2} \tag{13}$$

where κ depends solely on the parameters of the composite sector and can be complex only if the composite sector violates CP. When elementary fields couple to more than one state as in (6) or several invariants arise in the decomposition of $\mathbf{r}_L \times \mathbf{r}_R$, ΔX_μ will not be proportional to m_μ but will depend explicitly on the mixing parameters. In particular it can be complex even if the composite sector respects CP.

3 Results

We now apply the tools described in the previous section to the model given by the eqs. (4),(6). The relevant couplings of the muon to the heavy fermion resonances can be extracted from

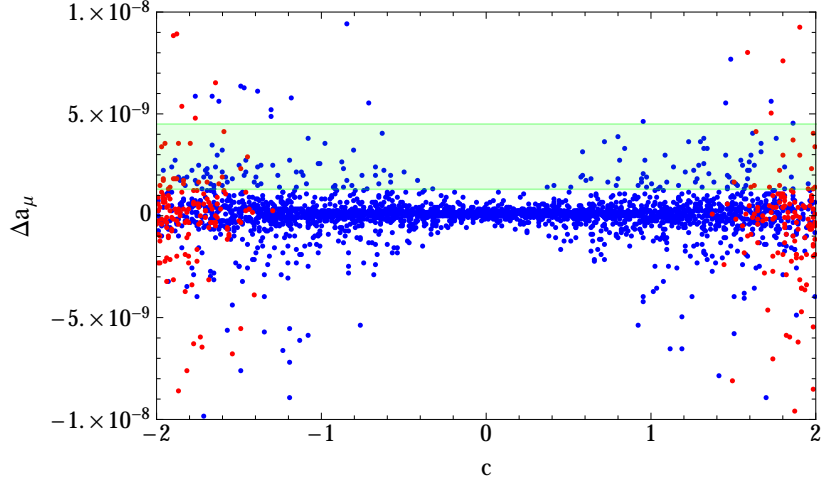


Figure 2: New physics contribution to Δa_μ for $c_L = c_R = c$ (real) and $f = 800$ GeV. The scan is performed by choosing $y \in [-0.1, 0.1]$ and $m_{1,4} \in [300, 3000]$ GeV. Blue points corresponds to fermionic contribution to the S parameter $\Delta S < 0.5$ assuming 3 degenerate generation partners (we use the formulas with finite terms of Ref. [13]). The green band represents the experimental value for Δa_μ within 2σ .

appendix A. Using the formulas above we find,

$$\begin{aligned}
\Delta X_\mu \simeq & \frac{m_\mu^2}{16\pi^2 f^2} + \frac{m_\mu}{16\pi^2} \left[\frac{1}{\sqrt{2}m_4} y_{L_4} y_{R_4} - \frac{c_L^*}{m_1} y_{L_1} y_{R_4} - \frac{c_R}{m_1} y_{L_4} y_{R_1} + \sqrt{2} \frac{c_L^* c_R m_4}{m_1^2} y_{L_1} y_{R_1} \right] s_h c_h \\
& + \frac{m_\mu}{24\pi^2} \left[\left(\frac{c_L}{m_4} - \frac{c_R}{m_1} \right) y_{L_4} y_{R_1} + \left(\frac{c_R^*}{m_4} - \frac{c_L^*}{m_1} \right) y_{L_1} y_{R_4} \right] s_h c_h \\
& + \frac{m_\mu}{24\sqrt{2}\pi^2} \left[\frac{m_4 c_R c_L^*}{m_1^2} y_{L_1} y_{R_1} - \frac{m_1 c_L c_R^*}{m_4^2} y_{L_4} y_{R_4} \right] s_h c_h
\end{aligned} \tag{14}$$

to leading order in the mixings. On the first line there are the contributions from non-derivative interactions mediated by the Higgs and the Z boson respectively. The contribution of W loops is zero due to a cancellation between the diagrams with doubly charged and neutral heavy fermion in the loop. In the second and third lines we show the contributions from the derivative Higgs interactions.

In Fig. 2 we plot a scan over the parameters of the model assuming real parameters and $c_L = c_R = c$. A sizable contribution to Δa_μ can be generated and the effect does not require extremely light fermions. Δa_μ tends to grow with c but larger values of c may lead to tension with bounds from S parameter, see discussion below. We should note that ΔX_μ is in general complex, even for real composite sector parameters. This implies strong bounds if a similar contribution is induced for the electron [14].

An interesting special case is obtained when the left and right chirality of the muon couple to a single operator of the strong sector. This can be realized for $y_{L_1} = y_{L_4}$ and $y_{R_1} = y_{R_4}$

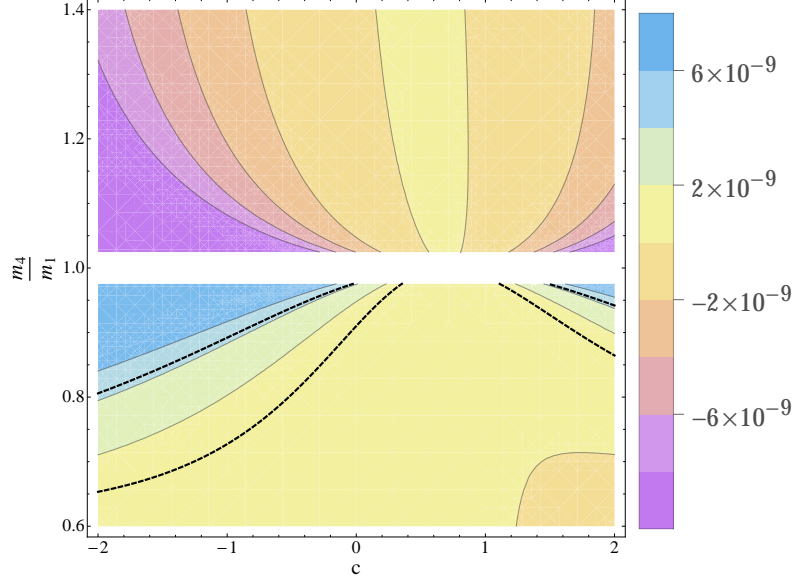


Figure 3: Contribution to Δa_μ in the scenario with $y_{L_1} = y_{L_4}$ and $y_{R_1} = y_{R_4}$ for $f = 800$ GeV. The 2σ experimental value is reproduced in the region between the dashes lines. The white horizontal strip corresponds to $y_{L,R} > 0.1$.

and it is the scenario effectively realised in extra-dimensional constructions (deviations from this relation correspond to non-minimal terms studied in [8]). For $c_L = c_R = c$ and real (CP and parity conserving composite sector) one finds,

$$\Delta X_\mu \simeq \frac{m_\mu^2}{16\pi^2 f^2} \left[1 + \frac{(m_1 - \sqrt{2} c m_4)^2}{m_1(m_1 - m_4)} + \frac{8}{3\sqrt{2}} c - \frac{2(m_1^2 + m_1 m_4 + m_4^2)}{3m_1 m_4} c^2 \right]. \quad (15)$$

As expected ΔX_μ is expressed in terms of the muon mass and composite sector parameters and it is real, contributing only to the magnetic dipole moment. In Fig. 3 we show a contour plot of Δa_μ as a function of m_4/m_1 and c . Δa_μ is enhanced for a small splitting between the quadruplet and singlet masses and grows with c . The light green region gives a contribution to Δa_μ in agreement with the experimental value at 1σ .

3.1 Bounds

The phenomenology of partially composite leptons was discussed in. [15], (see also [4]). Due to the smallness of their masses the compositeness of SM leptons is typically small leading to very mild constraints from modified couplings and compositeness bounds. For example the correction to the coupling of left-handed muons in the model discussed above is,

$$\frac{\delta g_{Z\mu_L\mu_L}}{g_{Z\mu_L\mu_L}^{SM}} \simeq -\frac{v^2}{1 - 2s_W^2} \left[\frac{y_{L_1}^2}{2m_1^2} + \frac{y_{L_4}^2}{2m_4^2} - \frac{\sqrt{2} c y_{L_1} y_{L_4}}{m_1 m_4} \right] \quad (16)$$

while the coupling of μ_R does not receive corrections at tree level. Large effect can only be obtained if some chirality of leptons are strongly composite. The most important indirect constraint arises from the S parameter. As we have seen the derivative coupling proportional to c is a key ingredient to obtain a sizable contribution to ΔX_μ , unless the resonances are almost degenerate. The same parameter also induces a calculable correction to S from loops of composite fermions [7, 13],

$$\Delta S \simeq \frac{2}{\pi} \frac{v^2}{f^2} (1 - 2c^2) \log \frac{\Lambda^2}{m_4^2} + \text{finite terms} \quad (17)$$

where Λ is an UV cutoff and finite terms depend on the regularization scheme. In the formula above we included a multiplicity factor for 3 generations. Indeed, realizing Minimal Flavor Violation (MFV) in these models requires a degenerate spectrum and couplings across different generations [16]. In Fig. 2 red points correspond to a fermionic contribution $\Delta S > 0.5$ and are therefore disfavoured from the experimental bound. Other contributions to S could however compensate this effect.

Direct searches from LHC exclude composite partners only up to 300-400 GeV. The most significant difference from other models of vector-like leptons concerns Higgs couplings. The mass spectrum and, as a consequence, the coupling of the Higgs to muons ($h_{\mu\mu}$) does not depend on $c_{L,R}$,

$$\frac{h_{\mu\mu}}{h_{\mu\mu}^{SM}} \simeq 1 - \frac{3}{2} \frac{v^2}{f^2}. \quad (18)$$

The modification of the Higgs coupling to fermions is in fact universal to leading order, depending only on the representation. With a phenomenologically plausible value $f = 800$ GeV or larger, $h_{\mu\mu}$ does not place a significant bound on our scenario. This removes the correlation between ΔX_μ and the Higgs couplings found in renormalizable models [2, 3]. In those Refs. the contribution to ΔX_μ needed to reproduce the experimental anomaly would imply an order 5-10 modification of the decay rate of the Higgs to muons, that is on the verge of being excluded by LHC measurements. Moreover in a complete flavor picture realising MFV an identical modification of the τ coupling to the Higgs would be generated that is grossly excluded by LHC measurements.

ΔX_μ in (14) is in general complex so that the imaginary part contributes to the muon EDM. When only two couplings exist the phase is different from zero if the composite sector violates CP ($c_{L,R}$ complex) and parity ($c_L \neq c_R$). At present this does not provide a constraint for the muon but an analogous contribution for the electron is tightly constrained [14]: the imaginary part should be suppressed by a factor 10^{-3} relative to Δa_e .

4 Discussion

In this note we computed the anomalous magnetic moment of the muon in theories with GB Higgs and partially composite fermions. Some new features arise compared to renormalizable theories studied in the literature. In particular, interactions associated to the GB

nature of the Higgs give extra contributions that can enhance Δa_μ and new diagrams with Higgs derivative interactions exist that can give a sizable effect. Our results show that it is plausible in certain regions of parameters to obtain a contribution that would account for the experimental anomaly. This depends crucially on the model dependent coupling c that controls the interactions of the Higgs with the composite fermions.

We should note that, working within a non-renormalizable effective field theory, our results should be interpreted as an estimate of the size of Δa_μ in this type of theories. Certainly we also expect UV contributions to the muon magnetic moment that are uncalculable in our framework. In particular composite sector operators such as¹,

$$\frac{1}{\Lambda} \bar{\Psi}_{4L}^i \sigma^{\mu\nu} \Psi_{4R}^j (T^a)_{ij} (f_{\mu\nu}^+)^a + h.c. \quad (19)$$

contribute to the magnetic moment of the muon. Assuming that dipoles are suppressed by a loop of the strong dynamics (as for example in weakly coupled 5D realizations of our framework) we find that their typical size is,

$$\Delta a_\mu^{UV} \sim \frac{1}{16\pi^2} \frac{m_\mu^2}{f^2} \quad (20)$$

which is an order of magnitude smaller than required to reproduce the anomaly for $f = 800$ GeV. The IR contribution from loops of light degrees of freedom would be in this case dominant. Nevertheless, we cannot a priori exclude that larger UV contributions are present.

It is interesting to cast our results into the broader flavor picture of partially composite Higgs models, see [15, 16] for a detailed discussion. The hypothesis of partial compositeness can suppress flavor transitions beyond the SM. Nevertheless, severe bounds exist especially in the lepton sector. For example $\text{Br}[\mu \rightarrow e\gamma] < 5 \times 10^{-13}$ hints to a scale of compositeness $\Lambda > 50$ TeV much larger than the value expected for these models if they are relevant to the hierarchy problem. Tension with flavor constraints can be eliminated if the theory realizes MFV. In fact, partial compositeness allows to elegantly realize this hypothesis: this requires that the composite sector possesses flavor symmetries that are only broken by mixings proportional to the SM Yukawa couplings. This can be realized if left-handed or right-handed fermions have equal degree of compositeness. One interesting prediction is that the contribution to the $(g - 2)$ of the electron is related to the one of the muon as,

$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \quad (21)$$

that could be of interest in future experiments [14]. Moreover contributions to EDMs are automatically zero at 1-loop if the strong sector also respects CP.

Our results can be extended in various directions. First, models with different representations of composite fermions or different patterns of symmetry breaking can be studied

¹We define $f_{\mu\nu} \equiv U^\dagger F_{\mu\nu} U = (f_{\mu\nu}^+)^a T^a + (f_{\mu\nu}^-)^{\hat{a}} T^{\hat{a}} \equiv f_{\mu\nu}^+ + f_{\mu\nu}^-$.

with the techniques described in this paper and other dipole moments relevant for composite Higgs models can be computed. One obvious generalization is for example the computation of chromo-magnetic operators in the quark sector. The same type of effects studied here also appears in models with extra-dimensions that correspond to an infinite number of resonances with derivative couplings determined by the metric. Finally the contribution of composite spin-1 resonances could also be studied along the lines described in this paper.

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A Relevant Formulas

In the CCWZ formalism one introduces the GB matrix,

$$U = e^{i\frac{\sqrt{2}}{f}\pi^{\hat{a}}T^{\hat{a}}} \quad (22)$$

where $T^{\hat{a}}$ are the broken generators, and constructs the Maurer-Cartan form

$$U^\dagger[A_\mu + i\partial_\mu]U = iU^\dagger D_\mu U = i d_\mu^{\hat{a}} T^{\hat{a}} + i e_\mu^a T^a. \quad (23)$$

Explicitly for $SO(5)/SO(4)$ this is given by,

$$\begin{aligned} d_\mu^{\hat{a}} &= \sqrt{2} \left(\frac{1}{f} - \frac{\sin \pi/f}{\pi} \right) \frac{\vec{\pi} \cdot D_\mu \vec{\pi}}{\pi^2} \pi^{\hat{a}} + \sqrt{2} \frac{\sin \pi/f}{\pi} D_\mu \pi^{\hat{a}} \\ e_\mu^a &= -A_\mu^a + 4i \frac{\sin^2(\pi/2f)}{\pi^2} \vec{\pi}^T t^a D_\mu \vec{\pi} \end{aligned} \quad (24)$$

with t^a the $SO(4)$ generators in 4x4 matrix form and

$$D_\mu \pi^{\hat{a}} = \partial_\mu \pi^{\hat{a}} - i A_\mu^a (t^a)^{\hat{a}}_{\hat{b}} \pi^{\hat{b}}. \quad (25)$$

For the model in section 2 the lagrangian can be written explicitly as,

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{kinetic} - (\bar{\Theta}_L \mathcal{M}_{-1} \Theta_R + \bar{\mathcal{N}}_L \mathcal{M}_N N_R + h.c.) - m_4 \bar{E}_{-2} E_{-2} \\ &+ \frac{g}{\sqrt{2}} [\bar{\mathcal{N}}_L g_L^{WN} \mathcal{W}^+ \Theta_L + \bar{\mathcal{N}}_R g_R^{WN} \mathcal{W}^+ \Theta_R + \bar{E}_{-2L} g_L^{WC} \mathcal{W}^- \Theta_L + \bar{E}_{-2R} g_R^{WC} \mathcal{W}^- \Theta_R + h.c.] \\ &+ \frac{g}{c_W} [\bar{\Theta}_L g_L^Z \mathcal{Z} \Theta_L + \bar{\Theta}_R^T g_R^Z \mathcal{Z} \Theta_R] + i \frac{c_L}{f} \bar{\Theta}_L \mathcal{R} \not{\partial} h \Theta_L + i \frac{c_R}{f} \bar{\Theta}_R \mathcal{R} \not{\partial} h \Theta_R \end{aligned} \quad (26)$$

where we have defined the fields,

$$\Theta_{L,R} = \begin{pmatrix} \mu \\ E \\ E_{-1} \\ \tilde{E} \end{pmatrix}_{L,R} \quad \mathcal{N}_L = \begin{pmatrix} \nu \\ N \end{pmatrix}_L \quad (27)$$

the mass matrices

$$\mathcal{M}_{-1} = \begin{pmatrix} 0 & y_{L_4} f \frac{1+c_h}{2} & y_{L_4} f \frac{1-c_h}{2} & y_{L_1} f \frac{s_h}{\sqrt{2}} \\ -y_{R_4} f \frac{s_h}{\sqrt{2}} & m_4 & 0 & 0 \\ y_{R_4} f \frac{s_h}{\sqrt{2}} & 0 & m_4 & 0 \\ y_{R_1} f c_h & 0 & 0 & m_1 \end{pmatrix} \quad \mathcal{M}_N = \begin{pmatrix} y_{L_4} f \\ m_4 \end{pmatrix} \quad (28)$$

and the couplings

$$\begin{aligned} g_L^Z &= \begin{pmatrix} -\frac{1}{2} + s_W^2 & 0 & 0 & 0 \\ 0 & -\frac{c_h}{2} + s_W^2 & 0 & -c_L \frac{s_h}{2} \\ 0 & 0 & \frac{c_h}{2} + s_W^2 & -c_L \frac{s_h}{2} \\ 0 & -c_L^* \frac{s_h}{2} & -c_L^* \frac{s_h}{2} & s_W^2 \end{pmatrix} \\ g_R^Z &= \begin{pmatrix} +s_W^2 & 0 & 0 & 0 \\ 0 & -\frac{c_h}{2} + s_W^2 & 0 & -c_R \frac{s_h}{2} \\ 0 & 0 & \frac{c_h}{2} + s_W^2 & -c_R \frac{s_h}{2} \\ 0 & -c_R^* \frac{s_h}{2} & -c_R^* \frac{s_h}{2} & s_W^2 \end{pmatrix} \\ g_L^{WN} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+c_h}{2} & \frac{1-c_h}{2} & c_L s_h \end{pmatrix} \quad g_R^{WN} = \begin{pmatrix} 0 & \frac{1+c_h}{2} & \frac{1-c_h}{2} & c_R s_h \end{pmatrix} \\ g_L^{WC} &= \begin{pmatrix} 0 & \frac{1-c_h}{2} & \frac{1+c_h}{2} & -c_L s_h \end{pmatrix} \quad g_R^{WC} = \begin{pmatrix} 0 & \frac{1-c_h}{2} & \frac{1+c_h}{2} & -c_R s_h \end{pmatrix} \\ \mathcal{R} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{pmatrix}. \end{aligned} \quad (29)$$

The relevant couplings used in the paper (denoted with a "hat") are obtained rotating to the physical mass basis defined by Eq. (28) (Higgs Yukawa couplings are given by $\lambda = d\mathcal{M}_{-1}/d\langle h \rangle$). Explicit formulae are easily derived to first order in the mixings sufficient for the analysis in this paper.

B Dipole Moments

In this appendix we present the relevant formulas for dipole moments in theories with GB Higgs. At 1-loop only states with charge -2, -1, 0 ($\chi_{-2,-1,0}$) contribute. We consider the following interaction terms,

$$\begin{aligned}\mathcal{L}_{int} = & [V_0^\mu g_L^{V_0} \bar{\mu}_L \gamma_\mu \chi_{-1L} + V_+^\mu g_L^{V_+} \bar{\mu}_L \gamma_\mu \chi_{-2L} + V_-^\mu g_L^{V_-} \bar{\mu}_L \gamma_\mu \chi_{0L} \\ & - \lambda_L \bar{\mu}_L h \chi_R + i \frac{C_L}{f} \bar{\mu}_L \not{\partial} h \chi_L + (L \rightarrow R)] + h.c.\end{aligned}\quad (30)$$

B.1 Non-derivative Interactions

With the couplings on the first line one finds the following contributions to the muon magnetic moment,

$$\begin{aligned}\Delta X_\mu^{V_0} &= \frac{m_\mu^2}{8\pi^2 m_{V_0}^2} \left[(|g_L^{V_0}|^2 + |g_R^{V_0}|^2) \mathcal{F}_{V_0}(x) + g_L^{V_0} (g_R^{V_0})^* \mathcal{G}_{V_0}(x) \frac{m_\chi}{m_\mu} \right] \\ \Delta X_\mu^{V_-} &= \frac{m_\mu^2}{16\pi^2 m_{V_-}^2} \left[(|g_L^{V_-}|^2 + |g_R^{V_-}|^2) \mathcal{F}_{V_-}(x) + g_L^{V_-} (g_R^{V_-})^* \mathcal{G}_{V_-}(x) \frac{m_\chi}{m_\mu} \right] \\ \Delta X_\mu^{V_+} &= \frac{m_\mu^2}{16\pi^2 m_{V_+}^2} \left[(|g_L^{V_+}|^2 + |g_R^{V_+}|^2) (4\mathcal{F}_{V_0}(x) + \mathcal{F}_{V_-}(x)) + g_L^{V_+} (g_R^{V_+})^* (4\mathcal{G}_{V_0}(x) + \mathcal{G}_{V_-}(x)) \frac{m_\chi}{m_\mu} \right] \\ \Delta X_\mu^h &= \frac{m_\mu^2}{16\pi^2 m_h^2} \left[(|\lambda_L|^2 + |\lambda_R|^2) \mathcal{F}_h(x) + \lambda_L \lambda_R^* \mathcal{G}_h(x) \frac{m_\chi}{m_\mu} \right]\end{aligned}\quad (31)$$

respectively for diagrams with V^0 , V^\pm and h in the loop. Here m_χ the mass of the heavy fermion. The loop functions are given by

$$\mathcal{F}_{V_0}(x) = \frac{-5x^4 + 14x^3 + 18x^2 \log x - 39x^2 + 38x - 8}{12(x-1)^4} \quad (32)$$

$$\mathcal{G}_{V_0}(x) = \frac{x^3 - 6x \log x + 3x - 4}{2(x-1)^3} \quad (33)$$

$$\mathcal{F}_{V_-}(x) = \frac{4x^4 + 18x^3 \log x - 49x^3 + 78x^2 - 43x + 10}{6(x-1)^4} \quad (34)$$

$$\mathcal{G}_{V_-}(x) = \frac{-x^3 - 6x^2 \log x + 12x^2 - 15x + 4}{(x-1)^3} \quad (35)$$

$$\mathcal{F}_h(x) = \frac{x^3 - 6x^2 + 6x \log x + 3x + 2}{6(x-1)^4} \quad (36)$$

$$\mathcal{G}_h(x) = \frac{x^2 - 4x + 2 \log x + 3}{(x-1)^3} \quad (37)$$

with $x = m_\chi^2/m_{V_0, V_\pm, h}^2$.

B.2 Derivative Interactions

The contribution of the diagram with two Higgs derivative interactions is formally given by,

$$\begin{aligned}
\Delta X_\mu^{(\partial h)^2} &\sim \int_0^1 u du \int \frac{d^4 l}{(2\pi)^4} \frac{A l^2 + B}{(l^2 - \Delta)^3} \\
A &= (2 - 3u) [m_\mu^2 (|C_L|^2 + |C_R|^2) - m_\mu m_\chi C_L C_R^*] \\
B &= 2 \{ [m_\mu^4 (u^2 - u^3) - m_\mu^2 m_\chi^2 u^2] (|C_L|^2 + |C_R|^2) - m_\mu^3 m_\chi u^3 C_L C_R^* \} \\
\Delta &= u(u - 1)m_\mu^2 + (1 - u)m_h^2 + u m_\chi^2
\end{aligned} \tag{38}$$

Naively the integral over momenta is logarithmically divergent and needs to be regularized. One can see that upon integration over u the result is finite but it depends on the regulator chosen. The different results correspond to the addition of UV local operators such as (19) to the effective action. For our estimates we perform the integral in 4D. Neglecting the muon mass relative to m_h and m_χ we find,

$$\begin{aligned}
\Delta X_\mu^{(\partial h)^2} &= -\frac{m_\mu^2}{16 \pi^2 f^2} \left[(|C_L|^2 + |C_R|^2) \mathcal{F}_{(\partial h)^2}(x) + C_L C_R^* \mathcal{G}_{(\partial h)^2}(x) \frac{m_\chi}{m_\mu} \right] \\
\mathcal{F}_{(\partial h)^2}(x) &= \frac{-2x^4 - 12x^3 + 6(2x - 1)x^2 \log x + 27x^2 - 16x + 3}{6(x - 1)^4} \\
&\quad + \frac{m_\mu^2}{m_\chi^2} \frac{3x^4 + (24x^4 - 12x^3) \log x + 10x^3 - 18x^2 + 6x - 1}{12(x - 1)^5} \\
\mathcal{G}_{(\partial h)^2}(x) &= \frac{2x^3 - 6x^2 \log x + 3x^2 - 6x + 1}{3(x - 1)^3} \\
&\quad - \frac{m_\mu^2}{m_\chi^2} \frac{2x^3 - 6x^2 \log x + 3x^2 - 6x + 1}{6(x - 1)^3}
\end{aligned} \tag{39}$$

The diagram with one derivative interaction and a Yukawa coupling has very similar features. In this case one finds,

$$\begin{aligned}
\Delta X_\mu^{\partial hh} &= -\frac{m_\mu}{16 \pi^2 f} [(C_L^* \lambda_L + C_R \lambda_R^*) \mathcal{F}_{\partial hh}(x) + (C_L \lambda_R^* - \lambda_L C_R^*) \mathcal{G}_{\partial hh}(x)] \\
\mathcal{F}_{\partial hh}(x) &= \frac{m_\mu}{m_\chi} \frac{6x^3 \log x - 11x^3 + 18x^2 - 9x + 2}{3(x - 1)^4} \\
\mathcal{G}_{\partial hh}(x) &= \frac{x^3 - 6x^2 \log x + 6x^2 - 9x + 2}{3(x - 1)^3} \\
&\quad + \frac{m_\mu^2}{m_\chi^2} \frac{7x^4 + 12(2x^4 - 2x^3 + x^2) \log x + 12x^3 - 36x^2 + 20x - 3}{12(x - 1)^5}
\end{aligned} \tag{40}$$

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